Questions and Sample Answers for Chapter 4

Question 1

What does the phrase “definitional grain size for attributes” mean? Why is it important to align the definitional grain size of attributes with the objectives of the diagnostic assessment and available data structures?

Question 2

What are some of the operational and theoretical advantages and disadvantages to postulating a specific attribute dependency structure in an attribute hierarchy?

Question 3

Describe the structure of a Q-matrix from both conceptual and statistical perspectives and its role for the specification of DCMs.

Question 4

The following *adjacency* matrix and *reachability* matrix are derived from a particular attribute hierarchy and can be used to derive a *reduced Q-matrix*. There are two parts to this question that concern these two matrices.

Part 1: Identify the adjacency matrix and the reachability matrix. Explain how you made your choice.

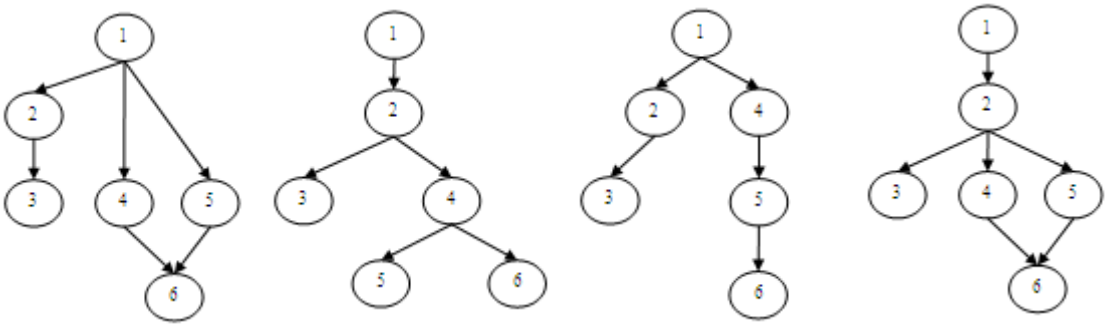
Matrix A

	Attribute 1	Attribute 2	Attribute 3	Attribute 4	Attribute 5	Attribute 6
Attribute 1	1	1	1	1	1	1
Attribute 2	0	1	1	1	1	1
Attribute 3	0	0	1	0	0	0
Attribute 4	0	0	0	1	0	1
Attribute 5	0	0	0	0	1	1
Attribute 6	0	0	0	0	0	1

Matrix B

	Attribute 1	Attribute 2	Attribute 3	Attribute 4	Attribute 5	Attribute 6
Attribute 1	0	1	0	0	0	0
Attribute 2	0	0	1	1	1	0
Attribute 3	0	0	0	0	0	0
Attribute 4	0	0	0	0	0	1
Attribute 5	0	0	0	0	0	1
Attribute 6	0	0	0	0	0	0

Part 2: Which of the following attribute hierarchies underlies the construction of the two matrices above? Explain how you made your choice.



(A) (B) (C) (D)

Question 5

The following matrix is an adjacency matrix that reflects a particular attribute hierarchy. Draw a graph of the attribute hierarchy. What is the number of attribute profiles that one would have to estimate under this hierarchy? What is difference between that number and the number of profiles if no hierarchy had been specified?

	Attribute1	Attribute2	Attribute3	Attribute4	Attribute5
Attribute 1	0	1	1	1	0
Attribute 2	0	0	0	0	1
Attribute 3	0	0	0	0	1
Attribute 4	0	0	0	0	1
Attribute 5	0	0	0	0	0

Section 2 – Sample Answers

Question 1

Within the framework of diagnostic measurement, the first step toward specifying DCMs is defining the learner characteristics / attributes about which we want to make diagnostic decisions and provide diagnostic feedback. Synonymous with the terms “latent trait” and “latent characteristic”, an attribute is defined as "the mental components that are of theoretical interest and are unobserved" (Rupp et al, 2010, p. 50). Domain-specific attributes vary in terms of the level of specificity at which they are defined. Degrees of specificity at which attributes are defines are referred to as the definitional grain size of the attributes, which can be metaphorically described as the theoretical resolution at which an investigator dissects a cognitive response process into its constituent components.

How attributes are defined and at what grain size relates closely to the objective of the diagnostic assessment, the psychological perspective that assessment specialists use in designing the assessment, and the data structures that arise from the implementation of the assessment. Assessment driven by a cognitive information-processing perspective frequently focus on delineating and segmenting relatively specific and narrow cognitive processes, and the mental components of those are defined as the attributes. Operating from the framework of behaviorism, diagnostic assessment can alternatively focus on specific behaviors that are indicative of a latent trait, a constellation of characteristic behaviors can be defined as an attribute. Generally speaking, most researchers would probably refer to as the first context as requiring a fine grain size of attribute definition and the latter context as requiring a coarse grain size of attribute definition.

Aligning the definitional grain size of attribute to the objective of the diagnostic assessment renders the design and implementation of the assessment meaningful and allows a researcher to derive the results that are needed for diagnostic decision making. Taking into consideration the developmental characteristics of the respondents who are the object of the assessment is also crucial for defining the definitional grain size of attributes as constructs like students' abilities vary across age groups and factors that drive the complexity of assessment tasks for one age group may not necessarily be the same for another age group.

Finally, even though it may be theoretically desirable to define a relatively large number of attributes – 10, say – it may not be feasible to estimate a parametric DCM without further constraints as the resulting number of attribute profiles / latent classes gets large quickly – 2^{10} = 1024 for example. In general, as the number of attributes is increased linearly (i.e., by adding more attributes), the number of attribute profiles increases exponentially (i.e., in a multiplicative fashion). Thus, determining an appropriate definitional grain size in a given application context requires a thoughtful compromise of theoretical and practical considerations.

Question 2

Specifying an attribute hierarchy offers a number of operational advantages. As the number of attributes to be modeled increases, the number of attribute profiles expected in the population increases exponentially. Postulating dependencies among the attributes significantly decreases the number of expected attribute profiles, thereby significantly reducing the required length of any instrument designed to distinguish between those profiles and reducing the parametric complexity of the DCM with which the data are analyzed. All else being equal, restricting the number of attribute profiles by specifying attribute dependencies will facilitate the estimation of models with more (and possibly more fine-grained) attributes. An additional theoretical advantage is that postulating a specific attribute hierarchy allows for the testing of specific hypotheses about the dependencies between attributes.

Although there are benefits to specifying an attribute hierarchy, there are also potential drawbacks. As part of a diagnostic system, the DCM should provide the diagnostic feedback that is necessary to support effective decision-making. If an assessment is specifically designed (based on a postulated attribute hierarchy) to empirically distinguish between only a subset of all possible attribute profiles, but the hierarchy was incorrectly specified or there are other profiles of interest in the population, it will not be possible to distinguish those profiles using the data collected during that assessment. The postulated path toward mastery described by the attribute hierarchy should be grounded both in theory and in empirical research with verbal protocol studies, verbal reports, results from eye-tracking studies, the input of a panel of experts, or some combination thereof supporting the hypothesized relationships between latent characteristics.

Question 3

Conceptually, the Q matrix is a specification of the attributes that are measured by a diagnostic assessment item. It is typically designed in the form of table with items in the rows and attributes in the columns, where ‘1’ and ‘0’ denote that an item measures a certain attribute or does not, respectively. From a statistical perspective, the Q matrix can be regard as the loading structure of a multidimensional factor model just as in confirmatory multidimensional factor analysis and item response theory models. Its major contribution for the development of diagnostic assessments is that it allows team members to understand how they design items that specifically target particular attributes.

Question 4

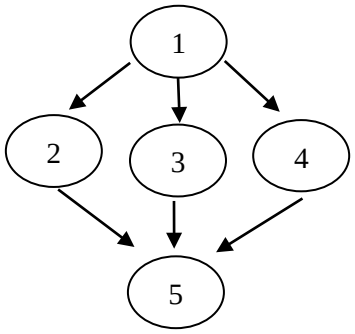
Part 1: An adjacency matrix only specifies direct relationships between attributes while a reachability matrix specifies both direct and indirect dependencies. Thus, the dependencies defined in the adjacency matrix should also be contained in the reachability matrix making matrix B the adjacency matrix and matrix A the reachability matrix. Another quick identification mechanism is an inspection of the diagonal elements, which are all 0 for an adjacency matrix as an attribute cannot be the direct predecessor of itself while they are all 1 in a reachability matrix as an attribute is always direct related with itself.

Part 2: Correct answer is d

According to the direct paths specified in the adjacency matrix, Attribute 2 is a direct and simultaneous predecessor of Attribute 3, 4, and 5; therefore, (d) presents the correct attribute hierarchy.

Question 5

The attribute hierarchy corresponding to the adjacency matrix is



All possible attribute profiles are listed in the following table:

	Attribute1	Attribute2	Attribute3	Attribute4	Attribute5
Profile 1	0	0	0	0	0
Profile 2	1	0	0	0	0
Profile 3	1	1	0	0	0
Profile 4	1	0	1	0	0
Profile 5	1	0	0	1	0
Profile 6	1	1	0	0	1
Profile 7	1	0	1	0	1
Profile 8	1	0	0	1	1
Profile 9	1	1	1	0	0
Profile 10	1	1	1	0	1
Profile 11	1	1	0	1	0
Profile 12	1	1	0	1	1
Profile 13	1	0	1	1	0
Profile 14	1	0	1	1	1
Profile 15	1	1	1	1	0
Profile 16	1	1	1	1	1

If the attributes are specified as being independent of each other (i.e., if no attribute hierarchy is specified), the number of all possible attribute profiles is 32. Therefore, the attribute hierarchy results in a reduction of 16 attribute profiles.