Questions and Sample Answers for Chapter 11

Section 1 – Sample Questions

Question 1

Describe the differences between the E-M algorithm and MCMC estimation for item parameters in DCMs in terms of their estimating procedures and assumptions.

Question 2

Consider a test consisting of three DINA items with Q matrix

Item	Attribute 1	Attribute 2
1	1	0
2	0	1
3	1	1

and starting parameter values

$P_0(\boldsymbol{\alpha}_1) = P_0(\boldsymbol{\alpha}_2) = P_0(\boldsymbol{\alpha}_3) = P_0(\boldsymbol{\alpha}_4) = 0.25$ and $s_{0,1} = s_{0,2} = s_{0,3} = g_{0,1} = g_{0,2} = g_{0,3} = 0.2$

where $\boldsymbol{\alpha}_1 = [0, 0]$, $\boldsymbol{\alpha}_2 = [0, 1]$, $\boldsymbol{\alpha}_3 = [1, 0]$, and $\boldsymbol{\alpha}_4 = [1, 1]$.

Compute the updated probability of each latent class membership after one expectation step of the E-M algorithm for a respondent with response pattern [1 ,0 ,1].

Question 3

Which of the following statements are true for item parameter estimation using the expectation-maximization (E-M) algorithm?

a. The E-M algorithm requires that the user determines the starting values for the expectation step. The user can either randomly assign starting values from a constrained distribution or empirically compute them from the data.

b. The expectation step is followed by the maximization step.

c. During the maximization step, the item parameters are estimated followed by structural parameter estimates. These are then re-estimated in an iterative process until the difference between parameter estimates from successive steps is minimal.

d. The efficiency of the E-M estimation is affected by the number of latent classes.

e. All of the above.

f. None of the above.

Question 4

Which of the following statements are true for the expectation-maximization (E-M) algorithm and which ones are true for the Markov-chain Monte Carlo (MCMC) estimation approach?

a. Using user-specified starting values increases the efficiency of the estimation approach.

b. The analyst is required to specify the formulas that yield posterior probabilities of latent class membership to make estimation work.

c. The complexity of the model to be estimated does not affect the efficiency and effectiveness of the estimation approach.

d. Subsequent parameter estimates computed during the estimation process are successively closer to the true parameter value under convergence of the estimation algorithm.

Question 1

The key difference between E-M algorithm and MCMC estimation in DCMs lies in the way how they estimate model parameters. The E-M algorithm is used under a frequentist estimation framework in which model parameters are viewed as fixed unknown quantities in the population; the objective is to obtain their point estimates and associated standard errors to allow for population inference. During estimation, the unknown attributes profiles are first replaced by expected values from a population distribution with known distribution values, item parameter estimates are derived from those, and the population distribution is updated in turn. This process is repeated until the changes between parameter estimates from successive iterations is very small (i.e., falls below a convergence threshold).

In contrast, the MCMC algorithm is used under a fully Bayesian estimation paradigm. The objective of the algorithm is to obtain a sample of values for each parameter from its respective posterior distribution without having to specify its functional form. This is done via successive draws from candidate distributions and under convergence this sequence of draws accurately represents the posterior distribution. In practice, researchers often discard a certain number of intermittent observations and only consider observations after an initial period known as burn-in.

Both estimation approaches can be used to specify posterior distributions for respondent parameters, but under the E-M algorithm this is accomplished simply via an application of Bayes' rule whereas those distributions are estimated (i.e., sampled from) directly using the MCMC algorithm. Nevertheless, under both estimation approaches researchers are interested in obtaining mean parameter values, known as expected a posteriori (EAP) estimates, and modal parameter values, known as maximum a posteriori (MAP) estimates.

Question 2

The numerators required for the probabilities of latent class membership are as follows:

Question 3

Correct answer: e

Question 4

a. This statement is true for both estimation approaches.

b. This is true only of the E-M estimation approach; in fact, MCMC estimation is designed for obtaining a sample of the posterior distribution without having to explicitly derive its mathematical expression / functional form.

c. This is false for both approaches, even though the effects are slightly different. MCMC estimation is sometimes argued to be more flexible than E-M estimation since even highly complex models can be estimated. However, the E-M algorithm is generally faster than the MCMC algorithm and often the preferred choice if it can be used. More complex models, which contain a larger number of parameters and more complex likelihood structures, are generally more challenging to estimate (i.e., estimation approaches become less efficient and, possibly, ineffective).

d. This is true for the E-M approach but not for the MCMC approach due to their differing objectives. The E-M approach is designed to "zero in on" the true parameter value in the population, which is why subsequent values are generally closer to this true value unless convergence issues are present. The objective of the MCMC algorithm is to obtain a representative sample of the posterior distribution, however, so that convergence here means convergence to a stationary distribution that is the posterior distribution. Therefore, the goal is that a set of generated numbers has this characteristic, some of which will come from the upper tail, some of which come from the lower tail, and some of which come from the center of the distribution.